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Algorithm for Autonomous Longitude and Eccentricity Control for Geostationary Spacecraft

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To lower satellite orbital maintenance cost, spacecraft designers are seeking flight software that provides more autonomy. Longitude and eccentricity are good candidates for autonomous control of geostationary spacecraft. The algorithm presented couples longitude control with eccentricity control. Longitude drift is modeled as one-degree-of-freedom motion and controlled with a quadratic equation predicting the subspacecraft Earth reference longitude after a predetermined amount of time. After formulation of the basic longitude control algorithm, addition of a differential corrections scheme resulted in an improved longitude error of $\pm 0.015^\circ$ longitude. Finally, implementation of longitude control and two-part maneuvers for eccentricity control successfully met the desired mission constraints. The algorithms developed form the basis for preparation of flight software for a geostationary spacecraft scheduled to launch in the next few years.

Introduction

IN 1945 Arthur C. Clarke proposed that long-range and even global communications would be possible using spacecraft in geostationary orbits.[1] The first geostationary Earth orbit (GEO) spacecraft was launched in 1963, with the number of GEO spacecraft climbing to more than 100 by 1980 and more than 200 by 1990.[2] The need and desire for these spacecraft has only increased. They are easy to locate with ground antennas, always stay in the same area in the sky, and require only one control node for orbit maintenance (although multiple downlink stations spaced around the Earth make orbit determination easier and better). Stationary (nontracking) antennas provide easy access to GEO spacecraft, thereby enabling many users with inexpensive antennas such as satellite television users. Communications is the primary use of GEO satellites, including both military and civilian applications, but there are a few scientific missions as well as Earth observing and Department of Defense reconnaissance missions.

To lower the cost of maintaining satellites in orbit, spacecraft designers are seeking flight software that provides more autonomy. Currently, ground crews perform orbit maintenance by determining the spacecraft orbit using some or all of the following: telemetry from the spacecraft, azimuth, elevation, and range and/or range rate from the ground antenna. If needed, orbit correction maneuvers are then calculated and executed by either real-time ground commands or delayed commands sent by the ground crew. Because the forces that perturb a GEO spacecraft's Earth reference longitude are understood and predictable, longitude station-keeping and orbit eccentricity control are good candidates for autonomous control. Eccentricity is typically adjusted at the same time as longitude, so that it is natural to combine these two corrections; moreover, every time a tangential or radial maneuver is performed, the eccentricity is affected.

The three-phase algorithm presented in this paper couples longitude control with eccentricity control. Initially, a plan for controlling longitude alone was developed and tested. After writing the basic longitude control algorithm, a differential corrections scheme was added using a more realistic model, improving longitude control from ± 0.04 to $\pm 0.015^\circ$ longitude. Because the point at which the maneuver is performed to control longitude is relatively unimportant, this maneuver was delayed until such a time in the orbit that it would not only correct the longitude error, but also help control eccentricity. This eccentricity control scheme (the second phase) is termed the sun-pointing-perigee strategy. Finally, longitude control and two-part maneuvers for eccentricity control were implemented (the third phase). The result is eccentricity control to the desired mission constraints, as well as effective longitude control that may be integrated with an autonomous station-keeping process.

The algorithm was originally written as an analysis tool to determine the amount of fuel required to maintain longitude station and eccentricity for GEO satellites. As a proof-of-concept exercise, the algorithm was configured and simulated as autonomous flight control software for a GEO satellite. This algorithm now forms the basis for autonomous east/west station-keeping and eccentricity control software modules to be flown on a GEO mission planned for launch in the next few years.

References 3–6 list a few of the many studies regarding station keeping of geostationary spacecraft. Shrivastava[3] provides a survey of station-keeping methods and the perturbation environment in GEO orbit. Chao and Baker[4] discuss propagation and station keeping of GEO spacecraft. Their approach uses separate maneuver to control longitude and eccentricity, whereas the algorithm described in this paper controls both with a single maneuver. Kamel et al.[5] describe a station-keeping method for a particular thruster configuration in which both xenon plasma and bipropellant thrusters are used and inclination and eccentricity are controlled by a single maneuver. Ely and Howell[6] use Poincaré sections in an algorithm to perform east/west station keeping on eccentric orbits. As in this current work, they use a differential corrections algorithm to compute station-keeping maneuvers. A key difference between the two control methods is the allowed time interval between maneuvers. Ely and Howell use a variable-time interval based on spacecraft drift violation of a defined deadband region. The current study uses a predetermined fixed-time interval because often operations schemes will require that maneuvers be performed at known time intervals.

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However, if desired, the algorithms presented in this work are easily adapted to a variable-time interval.

Basic Approach

Geostationary Station-Keeping Basics

The current longitude requirement for a commercial GEO satellite is on the order of $\pm 0.05°$. Since the orbit eccentricity is nonzero, as zero eccentricity is a mathematical abstraction, a daily longitude libration results. The instantaneous longitude error due to the orbit eccentricity is managed by controlling eccentricity. Because of this eccentricity-driven longitude libration, the longitude requirement often drives the eccentricity requirement. The maximum eccentricity permitted for a commercial GEO communication satellite is usually on the order of 10^{-4}.

In this study, a quasi-inertial right-handed frame is defined with its origin at the center of the Earth, the $\boldsymbol{x}$ axis aligned with the instantaneous vernal equinox direction and z axis directed toward the geographic north pole. This frame is a standard for GEO orbit analysis and is part of the mean equatorial geocentric system of date (MEGSD). In addition, a positive acceleration is assumed toward the east. A negative acceleration, or west acceleration, is analyzed with the same equations but opposite sign.

Longitude Control

When spacecraft referenced to earth longitudes are controlled, the longitude motion may be modeled as single-degree-of-frædom motion. (This may not be done at higher eccentricities because the longitude motion couples with longitude, semimajor axis, eccentricity, and tesseral effects.) It is assumed the orbit has sufficiently low inclination so that north/south (cross-track) motion can be ignored. Low eccentricity is also assumed so that radial motion can be ignored. (Effects of undesired components of station-keeping maneuvers along the radial direction are considered later.) In addition, the longitude "vibration" caused by nonzero eccentricity is ignored because the daily mean (not instantaneous) longitude is being controlled. The only other motion is in-track motion, also known as east/west drift. The spacecraft moves rapidly relative to inertial space but very slowly (ideally no motion) relative to Earth's surface. For longitude control, only east/west motion is monitored.

The primary natural force causing longitude acceleration is the Earth's nonuniform mass distribution giving rise to a nonuniform gravitational potential. This perturbative force comes from the tesseral terms of the Earth gravity model and is known as tesseral resonance. Tesseral resonance acts in the same way as a very low constant-magnitude tangential thrust. The acceleration magnitude and direction varies with longitude, which causes the drift time between burns to vary from station to station. The sign of the acceleration determines the direction of the acceleration (east being positive).

This problem is modeled using one degree of freedom because only longitude λ is of concern. (At higher eccentricities, other degrees of freedom must enter the analysis.) The spacecraft longitudinal acceleration is nearly constant because the spacecraft will remain in a relatively small neighborhood about the assigned longitude station, that is, $\Delta\lambda$ is small, so that $\ddot{\lambda}(\lambda, a)$ is nearly constant. With constant acceleration, the graph of longitude as a function of time is parabolically shaped. Figure 1 shows a desired longitude history with station keeping implemented when constant positive tesseral acceleration acts. The parabolic longitude history can be modeled as

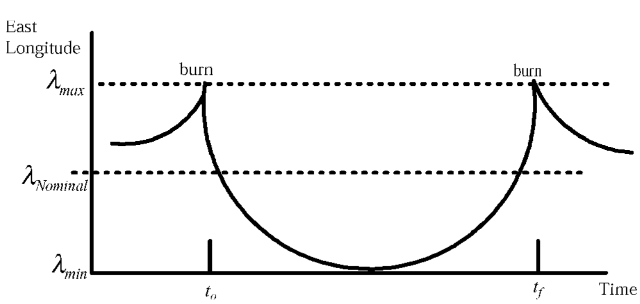


Fig. 1 Nominal longitude history.

$$\lambda(t) = \tfrac{1}{2}\ddot{\lambda}t^2 + \dot{\lambda}_0 t + \lambda_0 \tag{1}$$

where λ is the mean longitude ($\lambda \equiv M + \Omega + \omega - \theta_e$, where θ_e is the Earth mean Greenwich sidereal rotation angle), $\ddot{\lambda}$ is the longitude acceleration due to tesseral forces assumed constant, $\dot{\lambda}_0$ is the current longitude drift rate and λ_0 is the current longitude. Time t_0 is usually taken as zero for convenience, so that $\lambda_{\max}$ in Fig. 1 is equal to λ_0. Numerical values for $\ddot{\lambda}$ are available in Ref. 2.

Equation (1) is the primary equation used in the design of the following algorithm, which computes a tangential maneuver to return the spacecraft orbit to the appropriate location on Fig. 1. Equation (1) is rearranged to obtain initial drift rate in terms of initial position, final position, acceleration, and time as

$$\dot{\lambda}_0 = \left(\lambda_{\text{final}} - \lambda_0 - \tfrac{1}{2}\ddot{\lambda}t_f^2\right)\big/t_f \tag{2}$$

Ideally $\lambda_0 = \lambda_{\max}$ and $\lambda_{\text{final}} = \lambda_{\max}$; however, both angles will vary due to operational considerations, for example, a maneuver may be performed a day early if the nominal date falls on a holiday.

Once the desired drift rate $\dot{\lambda}_0$ is known, the current drift rate (established by orbit determination) can be subtracted from the desired drift rate to produce a delta drift rate $\Delta\dot{\lambda}$. The desired drift rate is not zero. It is the required drift rate to put the spacecraft on the path shown in Fig. 1. Accordingly, the station-keeping maneuver needs to change the drift rate by the amount

$$\Delta\dot{\lambda} = \dot{\lambda}_0 - \dot{\lambda}_{\text{current}} \tag{3}$$

To determine this maneuver, an equation is derived relating change in $\dot{\lambda}$ and ΔV. Begin with the energy equation

$$E = \tfrac{1}{2}v^2 - \mu/r = -\mu/2a \tag{4}$$

where v and r represent the spacecraft's speed and radius relative to Earth's center, μ is Earth's gravitational parameter, and a is the semimajor axis. Linearizing about a circular GEO orbit gives (by taking the differential)

$$v\,\mathrm{d}v = \tfrac{1}{2}(\mu/a^2)\,\mathrm{d}a \tag{5}$$

which can be rearranged as

$$\mathrm{d}a = 2(va^2/\mu)\,\mathrm{d}v \tag{6}$$

Now the velocity v and the mean motion n for a circular GEO orbit are

$$v = \sqrt{\mu/r} = \sqrt{\mu/a}, \qquad n = \sqrt{\mu/a^3}$$

Velocity v is substituted into Eq. (6) and simplified with mean motion n to give

$$\mathrm{d}a = 2\left[\sqrt{(\mu/a)}a^2\big/\mu\right]\mathrm{d}v = 2\left[\sqrt{(\mu a^4/a)}\big/\mu\right]\mathrm{d}v$$

$$= 2\left[\sqrt{\mu a^3}\big/\mu\right]\mathrm{d}v = 2\left[\sqrt{(a^3/\mu)}\right]\mathrm{d}v = (2/n)\,\mathrm{d}v$$

$$\therefore \mathrm{d}a = (2/n)\mathrm{d}v \tag{7}$$

Next, the period of a circular orbit is

$$P = 2\pi\sqrt{a^3/\mu} \Rightarrow 2\pi/P = \sqrt{\mu/a^3} \tag{8}$$

The drift rate D is the difference between the orbital period and Earth's rotation rate. A perfectly geosynchronous satellite has zero drift rate. The rotation rate of the Earth is subtracted from the mean motion to obtain drift rate as

$$D = \sqrt{\mu/a^3} - 2\pi/24\,\mathrm{h} \tag{9}$$

(A sidereal day should be used in a final analysis for improved precision.) Now the differential of D is taken and simplified using n as before (again assuming a circular orbit) to obtain

$$\mathrm{d}D = (-3/2)\sqrt{(\mu/a^5)}\,\mathrm{d}a = (-3/2)\left[\sqrt{(\mu/a^3)}\big/a\right]\mathrm{d}a$$
$$= (-3/2)(n/r)\,\mathrm{d}a$$
$$\therefore \mathrm{d}D = (-3/2)(n/r)\,\mathrm{d}a \tag{10}$$

Now combining Eqs. (6) and (10) gives

$$\mathrm{d}D = (-3/2)(n/r)(2/n)\,\mathrm{d}v \Rightarrow \mathrm{d}D = (-3/r)\,\mathrm{d}v \tag{11}$$

Drift rate D is equivalent to $\dot{\lambda}$ (rate of change of longitude), so Eq. (11) is rewritten in terms of $\dot{\lambda}$ and ΔV is substituted for dv to yield

$$\Delta\dot{\lambda} = (-3/r)\Delta V \tag{12}$$

Equation (12) can be solved for ΔV to get a ΔV in terms of $\Delta\dot{\lambda}$ by recalling the definition of $\Delta\dot{\lambda}$ from Eq. (3). Because this is a model of a GEO orbit, the radius of a GEO orbit replaces r in the final equation to give

$$\Delta V = (-r_{\mathrm{geo}}/3)\Delta\dot{\lambda} \tag{13}$$

(A nominal mean value for $r_{\mathrm{geo}} = 42{,}164.2$ km was selected and used throughout this study.) Equation (13) is used when estimating the magnitude of a tangential longitude station-keeping maneuver. The time between maneuvers and a maximum longitude limit are determined during mission design and used as inputs to the algorithm. When this predetermined time has passed since the last longitude station-keeping maneuver or the spacecraft longitude has reached the predetermined limit, Eq. (2) is used to determine a new desired drift rate as

$$\dot{\lambda}_{\mathrm{new}} = \left[\lambda_{\max} - \lambda_{\mathrm{current}} - \tfrac{1}{2}\ddot{\lambda}(\Delta t)^2\right]\big/\Delta t \tag{14}$$

(where Δt is the predetermined time between burns), and this new drift rate is used in Eq. (13) to calculate the appropriate tangential ΔV as

$$\Delta V_{\mathrm{tangential}} = (-r_{\mathrm{geo}}/3)(\dot{\lambda}_{\mathrm{new}} - \dot{\lambda}_{\mathrm{current}}) \tag{15}$$

Differential Corrections

There are many more forces, for example, lunar and solar gravitation, perturbing the longitude of a GEO orbit than those accounted for in the model used in the preceding section. All other perturbations are smaller than the tesseral forces. Instead of attempting to include these perturbations in the model when computing a station-keeping maneuver, experience demonstrated that it was more efficient to calculate a maneuver with the method described earlier (using the original, more simple model) and then improve it with differential corrections while using a more realistic model. The more realistic model used in this study included lunar and solar gravity models along with the tesseral model incorporating the following terms: C_{22}, S_{22}, C_{33}, S_{33}, C_{21}, S_{21}, and J_2. The solar and lunar locations were determined using equations taken from the *Astronomical Almanac*.[7,8]

In the formulation of the differential corrections algorithm, the ratio of the error in the final longitude $[\delta\lambda(t_f)]$ to the error in the ΔV ($\delta\Delta V$) is assumed to be the same as the ratio of a change in the final longitude $[\partial\lambda(t_f)]$ to a change in the ΔV ($\partial\Delta V$), or

$$\frac{\delta\lambda(t_f)}{\delta\Delta V} = \frac{\partial\lambda(t_f)}{\partial\Delta V}$$

thus,

$$\delta\Delta V = \frac{\partial\Delta V}{\partial\lambda(t_f)}\delta\lambda(t_f) \tag{16}$$

Here $\delta\lambda(t_f)$ is determined through simulation using a calculated ΔV from Eq. (15) and the more realistic model, but the $\partial\Delta V/\partial\lambda(t_f)$ ratio is found (approximately) analytically through the finite difference formula

$$\frac{\partial\Delta V}{\partial\lambda(t_f)} \approx \frac{\Delta V_2 - \Delta V_1}{\lambda_2(t_f) - \lambda_1(t_f)}$$

As shown in Appendix A, this finite difference may be approximated as $-r_{\mathrm{geo}}/3t_f$.

The differential corrections equation needed is then

$$\delta\Delta V = (-r_{\mathrm{geo}}/3t_f)\delta\lambda(t_f) \tag{17}$$

The approach to using differential corrections begins by calculation of a maneuver using Eq. (15). Then the orbit is propagated forward in time (using the more realistic model) with this maneuver until the next planned longitude correction. The difference between the simulated λ_{final} and the desired λ_{final} is computed. This difference is substituted in Eq. (17) for $\delta\lambda(t_f)$, and a correction to the maneuver is computed by adding $\delta\Delta V$ to the ΔV. One iteration was ample because additional iterations did not significantly improve the result.

Eccentricity Control

The eccentricity vector lies in the MEGSD equatorial plane and, therefore, can be defined by only two vector components. The ***ec*** unit vector points along the ***x*** axis of the MEGSD frame and the ***es*** unit vector points along the ***y*** axis. Mathematically the components of the eccentricity vector are defined as

$$ec = e\cos(\Omega + \omega) \tag{18}$$

$$es = e\sin(\Omega + \omega) \tag{19}$$

where $\Omega + \omega = \Pi$ is the longitude of perigee.

Now eccentricity can be analyzed and controlled as a vector. Because eccentricity is a vector, its control must consider both magnitude and direction.

Unlike longitude, which is negligibly affected by a small radial maneuver, eccentricity is affected by both tangential and radial maneuvers. The location of the spacecraft in the orbit when the maneuver takes place is also important. The effects on the eccentricity vector from a maneuver can be quantified as

$$\Delta\boldsymbol{e} = \langle\Delta ec,\ \Delta es\rangle$$

$$\Delta ec = \sin(S_b)(\Delta V_r/V) + 2\cos(S_b)(\Delta V_t/V) \tag{20}$$

$$\Delta es = -\cos(S_b)(\Delta V_r/V) + 2\sin(S_b)(\Delta V_t/V) \tag{21}$$

where S_b is the sidereal angle of the spacecraft at the time of the maneuver. [See Appendix B for a derivation of Eqs. (20) and (21).] Combining Eqs. (20) and (21) gives the magnitude of the eccentricity change as

$$\Delta e_{\mathrm{total}} = (1/V)\sqrt{4\Delta V_t^2 + \Delta V_r^2} \tag{22}$$

Left unmaintained, the eccentricity vector $\boldsymbol{e}$ will rotate about its origin once over the course of one year as shown in Fig. 2. This rotation of $\boldsymbol{e}$ is caused mainly by solar radiation pressure (SRP). When proper initial conditions (with perigee oriented toward the sun) are specified, the eccentricity magnitude will change very little throughout the year as the eccentricity vector rotates following the sun. The magnitude of eccentricity in this case may not be acceptable though. As stated earlier, the eccentricity requirement for a commercial geostationary communications satellite is usually on the order of 10^{-4}. If this requirement cannot be met with a passive approach, active control is required.

One simple method of mitigating the effect of the SRP perturbation is to perform longitude control maneuvers and even momentum unloading maneuvers at locations in the orbit where the resultant $\Delta\boldsymbol{V}$ will be opposite in direction of the perturbation caused by the SRP. Generally these types of maneuvers have a large tangential component. The energy already being spent to control longitude or momentum unloading is also used to control changes to eccentricity

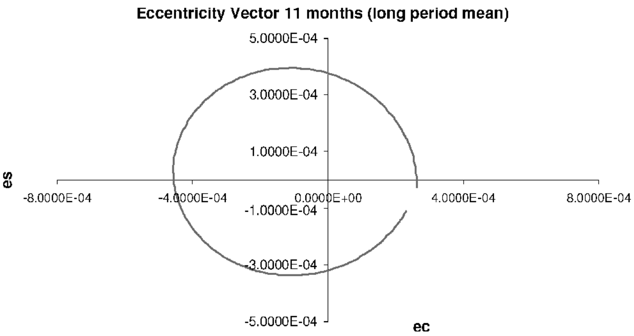
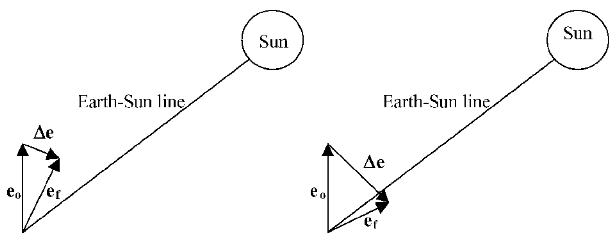


Fig. 2 Propagation of eccentricity over 11 months with no maintenance.



a) Less *e* available than desired b) More *e* available than desired

Fig. 3 Sun-pointing perigee method.

caused by SRP. If the maneuver is a prograde maneuver, or increases the velocity of the spacecraft, it is executed at 0600 hrs local satellite time (LST). A prograde maneuver executed at 0600 hrs LST will point in the opposite direction as the current perturbation caused by SRP. Retrograde maneuvers, or those decreasing the velocity of the spacecraft, are executed at 1800 hrs LST. This is an open-loop use of orbit maneuvers already being performed.

Another method of eccentricity control attempts to maintain the eccentricity vector pointing in the neighborhood of the Earth–sun line. This is known as a sun-pointing-perigæ strategy. This approach uses (only) the $\Delta \boldsymbol{V}$ from the longitude control maneuver and attempts to exploit the natural tendency of the eccentricity vector to follow the Earth–sun line. When longitude is controlled, the maneuver may be executed at any point on the orbit as long as the needed tangential ΔV magnitude, calculated in Eq. (15), is obtained. If the spacecraft thruster configuration is expected to produce an undesired radial component of ΔV along with the tangential ΔV, the radial component can be determined or estimated compared to the overall maneuver. The resulting magnitudes of the tangential and radial component $\Delta \boldsymbol{V}$ from a planned longitude control maneuver can be inserted into Eq. (22) to determine the magnitude of the $\Delta \boldsymbol{e}$ resulting from this longitude correction maneuver. Regardless of where in the orbit the maneuver is performed, the magnitude of $\Delta \boldsymbol{e}$ is determined by the magnitude of the longitude control maneuver. The approach from here is to plan the timing of the maneuver such that it will best benefit eccentricity control. The goal is to rotate $\boldsymbol{e}$ to lie just ahead of the Earth–sun line. If $\Delta \boldsymbol{e}$ is not long enough to rotate $\boldsymbol{e}$ all of the way back to the Earth–sun line, then $\boldsymbol{e}$ is rotated as far as possible toward the Earth–sun line without reducing the magnitude of $\boldsymbol{e}$. If there is more $\Delta \boldsymbol{e}$ than is required to rotate $\boldsymbol{e}$ ahead of the Earth–sun line, then the remaining $\Delta \boldsymbol{e}$ reduces the magnitude of $\boldsymbol{e}$ after rotating it. Figure 3 shows a demonstration of this strategy.

Note that this strategy combines the control of eccentricity and longitude, simplifying its implementation into an autonomous algorithm. Other approaches, such as the strategy presented by Chao and Baker,[4] control longitude and eccentricity using separate maneuvers. (Chao and Baker actually control the argument of perigee, which has the effect of limiting the eccentricity variation.) Because the focus in the current work was to create an autonomous control algorithm, ΔV optimization was not a prime concern. Comparing station-keeping costs to the Chao and Baker work is difficult because it is not clear if the two separate maneuver costs (longitude/eccentricity) in Chao and Baker's work can be added linearly to compare to the combined maneuver costs in this study.

Some missions require more precise handling of $\boldsymbol{e}$ than may be achievable using the approach described earlier. For these missions, two-part eccentricity maneuvers are required. A two-part maneuver can be planned to produce the desired longitude drift rate, the desired eccentricity magnitude, and the desired orientation of $\boldsymbol{e}$. A two-part maneuver is similar to a Hohmann transfer. Two maneuvers are performed spaced 180 deg apart in true anomaly on the orbit. Given sufficient fuel and adequate thruster configuration, $\boldsymbol{e}$ can be rotated to any orientation and magnitude. The first maneuver, executed at sidereal angle S_{b1}, adjusts perigee to the desired radius, whereas the second maneuver, executed at sidereal $S_{b2} = S_{b1} + \pi$, adjusts apogee. [The calculation of S_{b1} is shown later in Eq. (35).] These maneuvers, designated $\Delta \boldsymbol{V}_1$ and $\Delta \boldsymbol{V}_2$, result in eccentricity changes $\Delta \boldsymbol{e}_1$ and $\Delta \boldsymbol{e}_2$, respectively. They can also be performed in the opposite order as long as the total $\Delta \boldsymbol{e}$ desired is the sum of $\Delta \boldsymbol{e}_1$ and $\Delta \boldsymbol{e}_2$.

Once the current and desired orbital elements are calculated, the longitude and eccentricity corrections can be determined. The required $\Delta\dot{\lambda}$ can be calculated as detailed in the preceding sections and corrected with the differential corrections method. As stated earlier, the sidereal angle of this maneuver is not important for longitude control. It also does not matter (for the control of longitude) if the maneuver is performed in one or two parts. All that matters for longitude control is that the tangential components of the maneuvers performed sum to the right-hand side of Eq. (13) as

$$\Delta V_{\text{tangential sum}} = \Delta V_{1\text{tangential}} + \Delta V_{2\text{tangential}} = (-r_{\text{geo}}/3)\Delta\dot{\lambda} \tag{23}$$

The required $\Delta \boldsymbol{e}$ is simply found by subtracting the desired or final $\boldsymbol{e}$, which is determined during mission design, from the current $\boldsymbol{e}$,

$$\Delta \boldsymbol{e} = \langle \Delta ec,\ \Delta es \rangle = \boldsymbol{e}_{\text{final}} - \boldsymbol{e}_{\text{current}} \tag{24}$$

This gives both the magnitude and direction of $\Delta \boldsymbol{e}$. The right ascension of $\Delta \boldsymbol{e}$, needed later, is determined as the angle between the first point of Aries and $\Delta \boldsymbol{e}$ as

$$\Delta e_{\text{right ascension}} = \arctan(\Delta ec/\Delta es) \tag{25}$$

Note that S_{b1} and S_{b2} are π radians apart (since $S_{b2} = S_{b1} + \pi$ when performing a Hohmann-type transfer); the resulting $\Delta \boldsymbol{V}_1$ and $\Delta \boldsymbol{V}_2$ will either both be oriented along the spacecraft velocity or one may be opposite to the velocity. The directions depend on the required changes to apogee and perigee and, thus, the eccentricity vector. If both $\Delta \boldsymbol{e}_1$ and $\Delta \boldsymbol{e}_2$ are smaller in magnitude than $\Delta \boldsymbol{e}$, then $\Delta \boldsymbol{e}$, $\Delta \boldsymbol{e}_1$, and $\Delta \boldsymbol{e}_2$ are all parallel, that is, point in the same direction and, hence, have the same right ascension. If $\Delta \boldsymbol{e}_1$ is larger than $\Delta \boldsymbol{e}$, then $\Delta \boldsymbol{e}_1$ is parallel to $\Delta \boldsymbol{e}$, and $\Delta \boldsymbol{e}_2$ is antiparallel, that is, the right ascension of $\Delta \boldsymbol{e}_2$ is equal to the right ascension of $\Delta \boldsymbol{e}_1$ plus π radians. Also, if $\Delta \boldsymbol{e}_2$ is larger than $\Delta \boldsymbol{e}$, then $\Delta \boldsymbol{e}_2$ is parallel to $\Delta \boldsymbol{e}$ and $\Delta \boldsymbol{e}_1$ is antiparallel to $\Delta \boldsymbol{e}$. In all cases, the vector sum of $\Delta \boldsymbol{e}_1$ and $\Delta \boldsymbol{e}_2$ always equals $\Delta \boldsymbol{e}$.

If there are no radial components to the maneuvers, then the magnitude of $\Delta \boldsymbol{e}$ is [from Eqs. (20) and (21)]

$$\Delta e_{\text{tangential}} = (2/V)(\Delta V_{1\text{tangential}} - \Delta V_{2\text{tangential}}) \tag{26}$$

However, a radial component must usually be determined because some thruster configurations do not allow for a tangential maneuver without a radial component. Consider one maneuver at a time, and define $R_{\Delta e}$ as the ratio of the change in $\boldsymbol{e}$ due to both radial and tangential components of a single maneuver to the change in $\boldsymbol{e}$ due to the tangential component only,

$$R_{\Delta e} \triangleq \Delta e_{\text{total}}/\Delta e_{\text{tangential}} \tag{27}$$

From Eqs. (20–22) we know

$$\Delta e_{\text{total}} = \sqrt{\left(4\Delta V_{\text{tangential}}^2 + \Delta V_{\text{radial}}^2\right)}\Big/ V \tag{28}$$

$$\Delta e_{\text{tangential}} = 2\Delta V_{\text{tangential}}/V \tag{29}$$

Now combining Eqs. (27–29) gives

$$R_{\Delta e} = \sqrt{1 + (\Delta V_{\text{radial}}/2\Delta V_{\text{tangential}})^2} \tag{30}$$

An equation for Δe is known (magnitude only) taking into account both the tangential and radial components of both maneuvers by substituting Eq. (26) into Eq. (27) to give

$$\Delta e = (2R_{\Delta e}/V)(\Delta V_{1\text{tangential}} - \Delta V_{2\text{tangential}}) \tag{31}$$

Note this assumes that the same thruster (or thruster configuration) is used to execute both maneuvers so that $R_{\Delta e}$ is the same

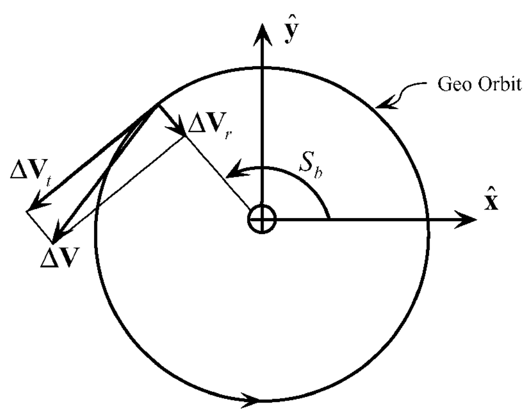


Fig. 4 Determination of maneuver location.

for each maneuver. Now recall Eq. (13) [expanded for clarity as Eq. (13a)],

$$\Delta V_{\text{tangential sum}} = \Delta V_{1\text{tangential}} + \Delta V_{2\text{tangential}} = (-r_{\text{geo}}/3)\Delta\dot{\lambda} \tag{13a}$$

giving two equations and two unknowns: Eqs. (13a) and (31) and unknowns $\Delta V_{1\,\text{tangential}}$ and $\Delta V_{2\,\text{tangential}}$. The ΔV can then be solved for as

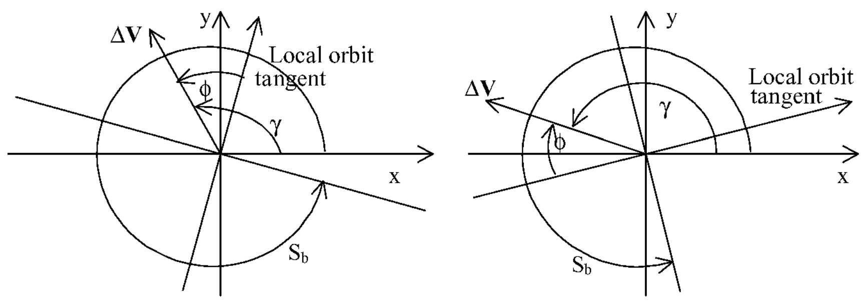
$$\Delta V_{1\,\text{tangential}} = V\Delta e/4R_{\Delta e} - (r_{\text{geo}}/6)\Delta\dot{\lambda} \tag{32}$$

$$\Delta V_{2\,\text{tangential}} = \Delta V_{\text{tangential sum}} - \Delta V_{1\,\text{tangential}} \tag{33}$$

At this point, the magnitudes and components of the maneuvers are known, but their locations are not yet determined. Equation (20) or (21) can be solved numerically to determine the locations. However, an analytic result can be found from the geometric argument illustrated in Fig. 4. The sidereal location of a maneuver can be determined by constructing a perpendicular line segment to the tangential component of the $\Delta\boldsymbol{V}$.

The $\Delta\boldsymbol{V}$ vector is first oriented using the angle between the $\Delta\boldsymbol{V}$ and the $\Delta\boldsymbol{e}$ vector. Refer back to Eqs. (20) and (21), and note that, as the sidereal angle of the maneuver changes, the magnitude of the resultant $\Delta\boldsymbol{e}$ does not change and neither does the relative orientation of $\Delta\boldsymbol{e}$ to $\Delta\boldsymbol{V}$. To determine the $\Delta\boldsymbol{V}$ orientation, the angle between $\Delta\boldsymbol{V}$ and $\Delta\boldsymbol{e}$ is needed. A simple case is shown and is easily generalized. With this information, the sidereal angles can be determined where the maneuvers should be executed to produce $\Delta\boldsymbol{e}$ in the desired direction.

Assume a ΔV with unity magnitude is executed at $S_b = 0$ that includes a small radial component directed inwardly. Let



a) Prograde maneuver **b) Retrograde manuever**

Fig. 5 Angles needed for determining the sidereal angle of a maneuver.

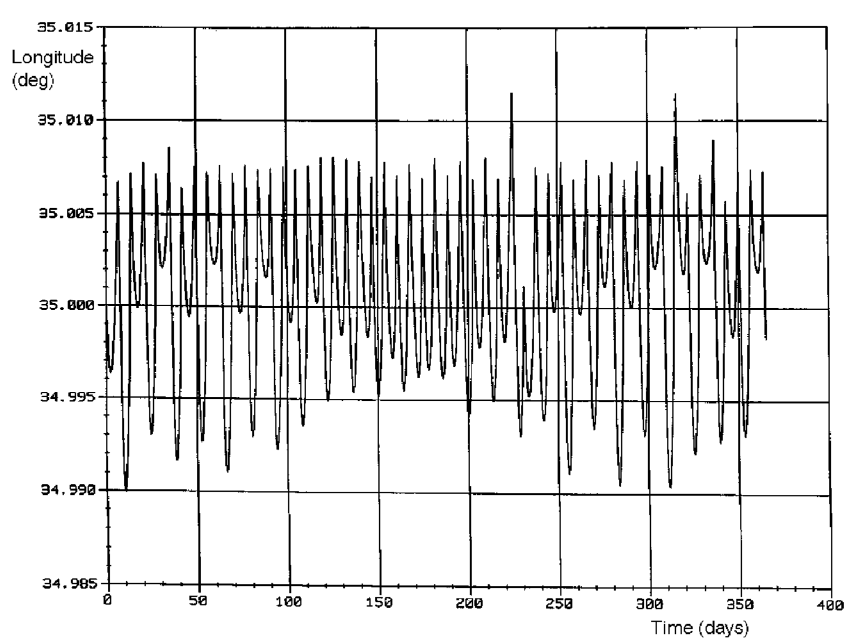


Fig. 6 Station keeping longitude history using eccentricity control.

$\Delta \boldsymbol{V} = \Delta \boldsymbol{V}_{\text{tangential}} + \Delta \boldsymbol{V}_{\text{radial}}$ where $|\Delta \boldsymbol{V}_{\text{tangential}}| = t$ and $|\Delta \boldsymbol{V}_{\text{radial}}| = r$. The sidereal angle of $\Delta \boldsymbol{V}$ is zero, and so from Eqs. (20) and (21)

$$\boldsymbol{e} = \langle \Delta ec,\ \Delta es \rangle = \langle 2t/V,\ -r/V \rangle, \qquad \boldsymbol{V} = \langle -r,\ t \rangle$$

Now the dot product is used with $\Delta \boldsymbol{V}$ and $\Delta \boldsymbol{e}$ to find the angle between the two vectors as

$$\Delta \boldsymbol{V} \cdot \Delta \boldsymbol{e} = |\Delta \boldsymbol{V}||\Delta \boldsymbol{e}| \cos\theta \Rightarrow \theta = \arccos\left[\frac{-3rt}{\sqrt{r^4 5r^2t^2 + 4t^4}}\right] \tag{34}$$

The right ascension of $\Delta \boldsymbol{e}$, calculated with Eq. (25) and the logic in the paragraph following Eq. (25), may now be added to θ from Eq. (34) to render the right ascension of $\Delta \boldsymbol{V}$, denoted γ. The angle γ is the measure of the angle from the first point of Aries to the $\Delta \boldsymbol{V}$ vector, not the location in the orbit where the maneuver is to take place. One more angle, ϕ, is required, that is, the angle between the $\Delta \boldsymbol{V}$ and its tangential component. (Recall that, due to the particular thruster configuration, a small radial component to the maneuver may occur.) Figure 5 shows the relationship between γ, ϕ, and S_b. The angle S_b is the sidereal angle of the maneuver and is calculated as follows.

For prograde maneuvers:

$$S_{b1} = \gamma - \phi + 3(\pi/2) \tag{35a}$$

For retrograde maneuvers:

$$S_{b1} = \gamma - \phi + (\pi/2) \tag{35b}$$

and S_{b2} is, as stated earlier, $S_{b1} + \pi$.

Results

Spacecraft parameters from a commercial communication satellite were used for this analysis. Initial orbit elements were chosen with zero inclination, Earth reference longitude of 35° east longitude, orbit epoch of 21 March 2006, eccentricity of 0.0003

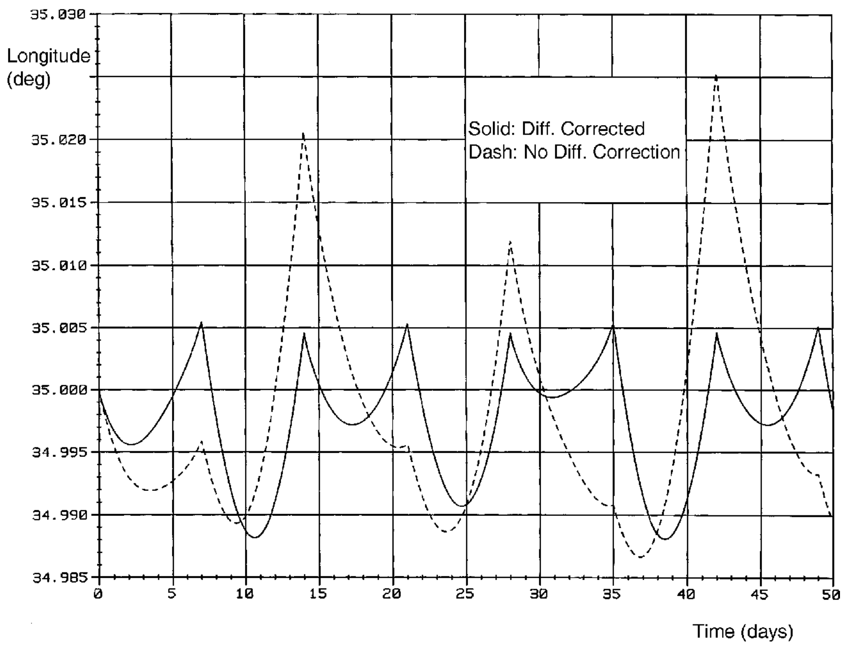


Fig. 7 Longitude history with and without differential corrections.

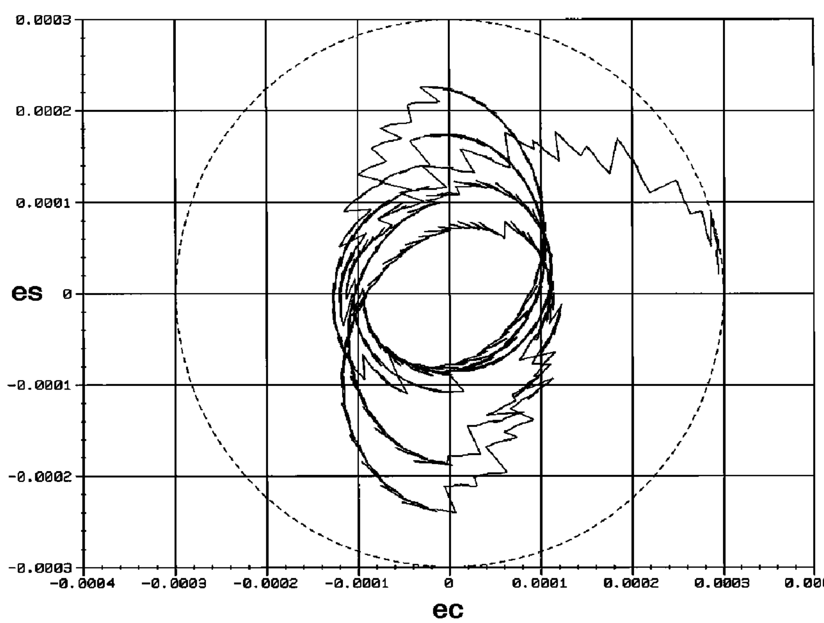


Fig. 8 Eccentricity history with sun-pointing perigee strategy employed.

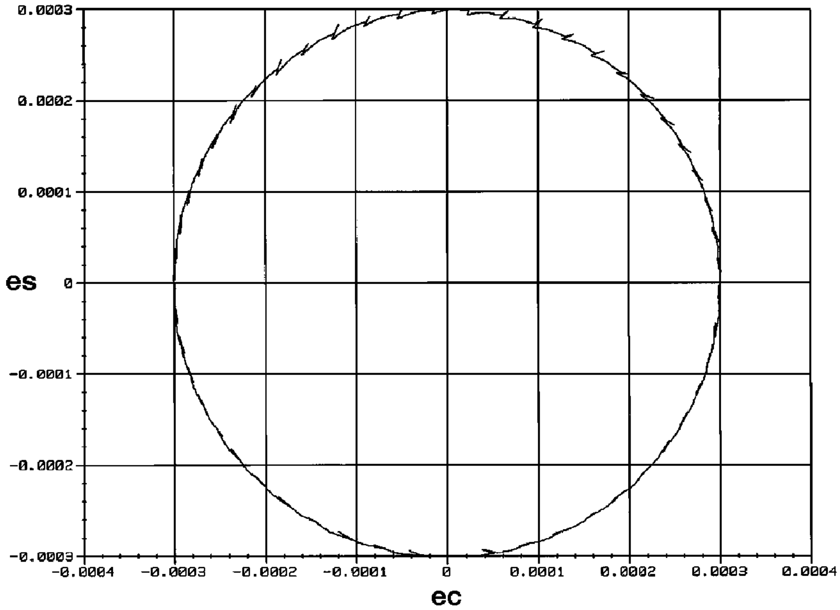


Fig. 9 Eccentricity history using two-part maneuvers.

and an argument of perigee of zero. Although testing and analysis were done with a variety of initial conditions, Figs. 6–9 refer only to the preceding specifications.

Figure 6 shows a longitude history of one year with a longitude station of 35° east longitude. Both longitude and eccentricity are controlled with single maneuvers [computed from Eqs. (14) and (15)] spaced at seven-day intervals. Note that the spacecraft longitude deviation never exceeds ±0.012 deg, well within the ±0.05 deg requirement.

Figure 7 shows two longitude histories. The broken line displays longitude control without differential corrections implemented, and the solid line includes differential corrections. Note the consistency of the longitude when the maneuvers are computed using differential corrections.

Figure 8 shows an eccentricity history over five years with the sun-pointing-perigee strategy employed. The axes of this graph are coincident with the MEGDS equatorial plane. The broken line circle shows the eccentricity limit of 0.0003. This circle is often referred to as an eccentricity control circle. Notice that the eccentricity vector makes five rotations about the origin, one per year, as it follows the Earth–sun line with the magnitude contained by the eccentricity control circle.

Figure 9 shows an eccentricity history over one year with two-part maneuvers. Like Fig. 8, the axes of this graph also coincide with the MEGDS equatorial plane. Each tic mark along the large circle results from the temporary change in eccentricity during the period between the two maneuvers. Notice that eccentricity is very tightly controlled. With two-part maneuvers, the eccentricity vector can be controlled to almost any desired magnitude and direction. With this control scheme, the eccentricity control circle does not have to be centered at the origin, as is required with some collocation schemes.

Conclusions

By the development of rules based on known orbit equations and how ground station crews currently control longitude and eccentricity, an algorithm has been written to perform the task of controlling longitude and eccentricity autonomously aboard a spacecraft. The algorithms developed form the basis for flight software being prepared for a geostationary spacecraft scheduled to launch in the next few years.

Inclination is an orbit parameter that has not been controlled in this algorithm. It is one area of possible future research. Additionally, this algorithm assumes the use of instantaneous ΔV. Because continuous/low thrust would likely require a different approach, future work could develop modifications to the methods presented here when continuous thrusting systems are used.

Appendix A: Derivation of Differential Corrections Sensitivity Term

An analytic approximation for the $\partial\lambda(t_f)/\partial\Delta V$ term is used in the differential corrections algorithm. Begin with the finite difference formula

$$\frac{\partial\lambda(t_f)}{\partial\Delta V} \approx \frac{\lambda_2(t_f) - \lambda_1(t_f)}{\Delta V_2 - \Delta V_1} \tag{A1}$$

then the difference in λ_1 and λ_2 is computed using Eq. (1) with the initial drift rate perturbed by $\Delta\dot{\lambda}$ to give

$$\lambda\,(t) = \tfrac{1}{2}\ddot{\lambda}t^2 + (\dot{\lambda}_0 + \Delta\dot{\lambda})\,t + \lambda_0 \tag{A2}$$

Recalling Eq. (12) and substituting in Eq. (A2) gives

$$\lambda_1(t_f) = \tfrac{1}{2}\ddot{\lambda}t^2 + [\dot{\lambda}_0 - (3/r_{\text{geo}})\Delta V_1]t + \lambda_0 \tag{A3}$$

Defining

$$\Delta V_2 = \Delta V_1 + \delta \tag{A4}$$

and replacing ΔV_1 in Eq. (A3) with ΔV_2 gives $\lambda_2(t_f)$ as

$$\lambda_2(t_f) = \tfrac{1}{2}\ddot{\lambda}t_f^2 + [\dot{\lambda}_0 - (3/r_{\text{geo}})(\Delta V_1 + \delta)]t_f + \lambda_0 \tag{A5}$$

Subtracting Eq. (A3) from Eq. (A5) gives

$$\lambda_2(t_f) - \lambda_1(t_f) = -(3/r_{\text{geo}})\delta \cdot t_f \tag{A6}$$

From Eq. (A4)

$$\Delta V_2 - \Delta V_1 = \delta \tag{A7}$$

thus,

$$\frac{\partial\lambda(t_f)}{\partial\Delta V} = \frac{-3t_f}{r_{\text{geo}}} \tag{A8}$$

Hence, the differential correction equation needed is

$$\delta\Delta V = (-r_{\text{geo}}/3t_f)\delta\lambda(t_f) \tag{A9}$$

Appendix B: Derivation of the Effects on the Eccentricity Vector from the Application of a Small Maneuver

The instantaneous change in the eccentricity vector due to a small $\Delta \boldsymbol{V}$ applied along a geostationary orbit can be found by taking the differential of the eccentricity vector

$$\boldsymbol{e} = (1/\mu)(\boldsymbol{V} \times \boldsymbol{h}) - \hat{\boldsymbol{r}} \tag{B1}$$

to obtain

$$\delta \boldsymbol{e} = (1/\mu)(\delta \boldsymbol{V} \times \boldsymbol{h} + \boldsymbol{V} \times \delta \boldsymbol{h}) - \delta \hat{\boldsymbol{r}} \tag{B2}$$

where $\boldsymbol{e}$ represents the eccentricity vector, μ is the gravitational parameter, $\boldsymbol{V}$ is the velocity of the spacecraft, $\boldsymbol{h}$ is the specific angular momentum of the spacecraft, and $\hat{\boldsymbol{r}}$ is the unit vector along the radial direction. Because the location of the spacecraft does not change during the application of the instantaneous maneuver, $\delta \hat{\boldsymbol{r}} = 0$.

To further simplify Eq. (B2), the direction perpendicular to $\hat{\boldsymbol{r}}$ within the orbit plane and in the general direction of the spacecraft velocity is defined as $\hat{\boldsymbol{\theta}}$, whereas the direction along the angular momentum vector is defined as $\hat{\boldsymbol{h}}$ to complete the right-handed rotating frame. Then the position vector of the spacecraft is defined as

$$\boldsymbol{r} = r\hat{\boldsymbol{r}} \tag{B3}$$

whereas the velocity is designated

$$\boldsymbol{V} = V_r \hat{\boldsymbol{r}} + V_t \hat{\boldsymbol{\theta}} \tag{B4}$$

Substituting Eqs. (B3) and (B4) into Eq. (B2) and recalling that $\boldsymbol{h} = \boldsymbol{r} \times \boldsymbol{V}$ so that $\delta \boldsymbol{h} = \boldsymbol{r} \times \delta \boldsymbol{V}$ (applying $\delta \hat{\boldsymbol{r}} = 0$ again) results in

$$\delta \boldsymbol{e} = (r/\mu)[(2V_t \Delta V_t)\hat{\boldsymbol{r}} - (V_t \Delta V_r + V_r \Delta V_t)\hat{\boldsymbol{\theta}}] \tag{B5}$$

The unit vectors $\hat{\boldsymbol{r}}$ and $\hat{\boldsymbol{\theta}}$ can be expressed in terms of the MEGSD frame using the sidereal angle of the spacecraft at the time of the maneuver as

$$\begin{aligned} \hat{\boldsymbol{r}} &= \cos S_b \hat{\boldsymbol{x}} + \sin S_b \hat{\boldsymbol{y}} \\ \hat{\boldsymbol{\theta}} &= -\sin S_b \hat{\boldsymbol{x}} + \cos S_b \hat{\boldsymbol{y}} \end{aligned} \tag{B6}$$

Substituting Eq. (B6) into Eq. (B5) produces

$$\begin{aligned} \delta \boldsymbol{e} = (r/\mu)[&(2V_t \Delta V_t \cos S_b + V_t \Delta V_r \sin S_b + V_r \Delta V_t \sin S_b)\hat{\boldsymbol{x}} \\ &+ (2V_t \Delta V_t \sin S_b - V_t \Delta V_r \cos S_b - V_r \Delta V_t \cos S_b)\hat{\boldsymbol{y}}] \end{aligned} \tag{B7}$$

For a geostationary orbit, V_r is small and can be ignored to first order. Similarly, V_t is approximately equal to V and $r/\mu \cong 1/V^2$ for small eccentricities. Using these approximations in Eq. (B7) gives the final result as

$$\begin{aligned} \delta \boldsymbol{e} = &[2(\Delta V_t / V) \cos S_b + (\Delta V_r / V) \sin S_b]\hat{\boldsymbol{x}} \\ &+ [2(\Delta V_t / V) \sin S_b - (\Delta V_r / V) \cos S_b]\hat{\boldsymbol{y}} \end{aligned} \tag{B8}$$

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